Potential collapse of whitespaces and the prospect for a universal power rule

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***Abstract*—The TV whitespaces have recently been opened up for semi-unlicensed use by frequency-agile radios. However, there is a potentially significant flaw in the adopted rules: they try to treat the whitespaces in a manner similar to the ISM bands — with per-device transmit-power constraints. Unfortunately, wireless interference aggregates and the population density across the United States of America varies by orders of magnitude. This means that the aggregate interference that TV receivers might face could increase as whitespace devices are deployed, and could *collectively* cause a loss of reception within the supposedly protected contours. However, it is not too late. The adopted geolocation plus databases approach lets us avoid this problem by changing database behavior — instead of just controlling where white-space devices operate, we should also hold their aggregate emissions to within a certain power density (*i.e.* by area).**

With the looming problem resolved, we can also try to address one of the main tensions within the entire TV whitespace approach: any set of allowed power/height/distance rules is implicitly prioritizing rural vs urban needs and picking favorites among different applications. Alas, the reality of aggregate interference prevents us from making everyone simultaneously perfectly happy. To enable higher transmit powers further from TV stations, we must necessarily reduce the allowed powers closer in. But amazingly, the properties of wireless propagation and information-theory combine to suggest that universally approximately-optimal approaches might be possible that could compromise between these competing interests in a principled way. We explore a pair of such rules and show that indeed, most people can get a data-rate close to what they would have gotten if the rules had been written especially for them.

I. Introduction

The television whitespaces represent the first major test of wireless coexistence among heterogenous uses that are not participating within the same system, where at least one of the uses is deserving of a strong quality-of-service guarantee. In November 2008, the FCC released the initial version of rules [1] designed to allow the operation of unlicensed non-TV devices within the TV whitespaces, and these rules were esssentially confirmed and further clarified in September 2010 [2]. The rules are predicated on three main ideas:

- Defining zones around each TV transmitter that are deemed protected. The idea is that whatever additional interference occurs due to the transmissions of legal TV whitespace devices, it should not be enough to cause TV receivers within the protected zones to lose service.
- Regulating the transmissions coming from whitespace devices in a lighthanded manner analogous to unlicensed devices operating in the ISM bands: they have individual power masks that they must meet and limits on their antenna heights.
- Dealing with the issue of protecting TV receivers by mandating that whitespace devices figure out roughly where they are and check with an authorized database to make sure that it would be safe for them to operate in this location. The particular safety check envisioned in the rules involves a simple check to make sure that they are far enough (e.g. 14.4 km separation margin for co-channel) away from the protected zones.

These three ideas are simple, and there is nothing obviously wrong with any of them. However, each one implicitly requires the regulators to make some policy choices — we must decide where the protected zones are, what power masks and antenna heights to permit, and finally, what constitutes the safety check in the database. They are also not all conceptually equal in status. The first one is about expressing what we mean by protecting the primary users while the subsequent choices are meant to respect that choice, even if as a matter of formal rule-making they each stand on their own.

In this paper, we confront two core problems:

1) The *physical fact* that wireless interference aggregates and that this aggregation is more important for wireless signals that propagate well.
2) The seemingly arbitrary nature of the political choices that must be made in determining the individual power-masks and safe separation margins.

Of these, we will see that (1) is a looming issue that must be addressed, and hopefully soon. If multiple whitespace devices are active simultaneously — as they will be if they are commercially successful — then under the current interpretation of the rules, we believe harmful interference might occur to protected TV receivers. The issue of aggregate interference in the TV whitespace context was first raised in [3], [4], and the community has worked on technical approaches to quantifying it since then (e.g. [5]).

We argue in this paper that just tweaking the numerical parameters in the existing rules to fix this problem is undesirable because the population density across the USA is so wildly variable. Furthermore, we see that in principle, just relying on the "wild" media-access-control (MAC) protocols within wireless standards may not be enough to deal with this problem. Fortunately, we show that within the database approach there is another way to move forward. The key idea we propose is to move the power constraint out of the individual device and make it something that must be satisfied by the *collective* sea of devices in operation at any given place at any given time — to regulate the local *power density* rather than the individual powers. Although working out exactly how to do this is beyond the scope of this particular paper, back-of-the-envelope calculations suggest strongly that such power-density constraints can be accommodated by "tame" MAC protocols in a natural scaling-friendly way. The databases must inform devices of their locally permitted power density.

(2) is a longer-term issue that addresses a seemingly intractable and fundamentally political tradeoff. Rural users can argue that they are far away from the protected zones and so should be allowed to transmit at higher powers, thereby enabling them to achieve connectivity among their more widely scattered dwellings. By contrast, urban users could be satisfied by lower power signals, but would like to use them even closer to the protected zones so that they can gain access to more spectrum. There is no way to give them both exactly what they would want, and moreover, there is no single sense of "rural" or "urban." Instead, there is a continuum of possible users, each of whom might have his own preferred combination of a permitted transmit power and required safety separation. We will see that the roughly inverse-power-law nature of wireless propagation combined with the fundamentally logarithmic nature of information-theoretic channel capacity allows us to envision universal rules that give almost everyone a good approximation[1] of what they could have gotten if they had won the political battle. This is more in the spirit of "light-handed regulation" since those technical approaches that are best situated will more or less continue to be best situated, and the economic competitive landscape will not be unduly distorted by the political choices that must be made to allow anything at all to happen.

The story told in this paper deals with two different application models for the TV whitespaces, both of which measure the quality of whitespace rules by the resulting data rates that are supportable. One application model is inspired by the "WhiteFi" model [12] — we conceptualize this with a toy model of deploying whitespace WiFi-style access points to create local hotspots. We assume these hotspots have a range of about 100 meters, although the results will clearly be qualitatively similar if this range were either lengthened or shortened. But the other application model that we consider, and in fact spend more time discussing in detail, is one in which the TV whitespaces are used to support the air-interface of a cellular-style system that aims for nearly universal coverage. We believe that this cellular-style model is important to study for many reasons:

- It allows us to roughly evaluate whether or not the TV whitespaces might enable disruptive innovation that will allow a new entrant to potentially challenge the legacy wireless carriers without having to obtain licensed spectrum in an auction or in a secondary market. After all, the propagation characteristics within the TV band are as good as or superior to those of the existing cellular bands. Meanwhile, the ISM bands are worse propagation-wise. Even the possibility of such a disruptive entry could change the balance of power between different commercial players within the wireless space.
- The cellular providers and their equipment vendors themselves are actively studying the potential of using the TV whitespaces to augment the performance of their own networks.
- A wireless ISP is arguably more like a cellular provider than a hotspot provider. The wireless ISP wants to be able to provide service to residents in their geographically spread-out homes and not in the vicinity of a single gathering place like a hotspot.
- It provides us with a concrete point of contrast to the hotspot model that lets us explore whether the application model makes any difference.

For both the cellular-inspired and hotspot-inspired models, we assume that the density of access-points or cell towers is proportional to the population density. For the hotspot-inspired model, this is reasonable due to finite market-penetration within the home market combined with the indirect effect that the gathering-places deploying non-home hotspots require some population catchment of customers/taxpayers to be economically viable. For the cellular-inspired models, this is because the cost of the cell tower must be amortized over enough paying customers to justify operating that tower.

We hope that by exploring these issues quantitatively within the concrete setting of the United States' TV whitespaces, and by building upon our earlier work on estimating whitespace quality [13], that it will be easier for a wider audience to engage with the issues discussed here. Unless noted, all of our methodology regarding our idealizations of the FCC rules, wireless propagation models, and population densities follows what we had done earlier in [14].

II. What is wrong with the current FCC rules

The implicit idea behind the form of the current rules is that the value for the geographic separation margin is chosen such that a whitespace device operating at such a "safe" distance from a primary receiver would cause no more than X dB of interference. For example $X = 3$ corresponds to saying that the wireless interference from this one device is at the same level as thermal noise. Choosing any value $X < 3$ allows one to comfortably say that the resulting interference is "less than thermal noise." While in English this seems to be almost the same as saying that it is negligible, that is not necessarily a safe assumption. One fire ant is not dangerous, but thousands of fire ants together are not.

[1] The idea of universal approximations has been the dominant approach to tackling otherwise computationally intractable problems in the Theoretical Computer Science literature, and has recently become a hot topic within information theory as well. Recently, it has allowed for conceptual breakthroughs in our understanding of the wireless interference channel [6], the issue of wireless relaying [7], and on a classic problem in decentralized stochastic control theory dealing with "signaling" called the Witsenhausen Counterexample [8]. Here, the goal is to distill this theoretical inspiration into more practical advice — when it isn't possible to give everyone everything, the right thing to do is to bound how much everyone loses over what they could've gotten if they had won over all other competing interests. This is intimately related to the whole area of "fairness" in networking [9]–[11].

The challenge comes from the very feature of the TV bands that makes them so attractive — their good propagation characteristics. Viewed from 15 km away, all the users within an entire square-kilometer are about equally far away. However, a square-kilometer might contain a great many (hundreds to thousands) of wireless devices (think of a university campus if you have any doubt).

To see the effect quantitatively, consider a toy model in which we have a 500m tall TV tower operating on channel 21 with 100kW, secondary users at 1W each and a population density of 429 people/sq km — the median across the United States of America. Suppose that one whitespace device is simultaneously active for every 500 people in the country and these devices respect the 14.4 km separation distance mandated for fixed devices. Intuitively, this might not raise any red flags, but as shown in Figure 1, the effectively protected radius shrinks from the intended 108.5km to 97.4 km. People who might have felt that they were safely 10km within the protected contour could lose TV service.[2] Clearly the rules that are sufficient in the one-secondary case can be lacking when we anticipate more secondary users.

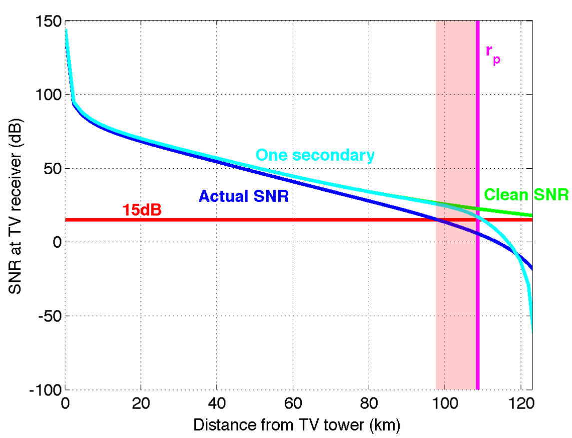

Fig. 1. r_p drops from 108.5km to 97.4km with the addition of secondary transmitters. TV receivers at the specified r_p are safe in the presence of a single secondary user, but not surrounded by a sea of them.

One might complain that this uniform population density model is too much of a toy. So instead, we ran a scenario where the TV whitespaces are used to support WiFi-style wireless systems with one fixed access-point active for every p people. Alternatively, you can view this as a cellular (or wireless ISP) model with one tower for every p people. From the perspective of aggregate interference it makes very little difference. We assume that each tower is transmitting at 1W and with a height of 30m. We can see the overall effect of this aggregate interference in Figures 2 and 3. As should be expected, the number of protected channels whose reception is lost increases as the market penetration of active whitespace devices increases.

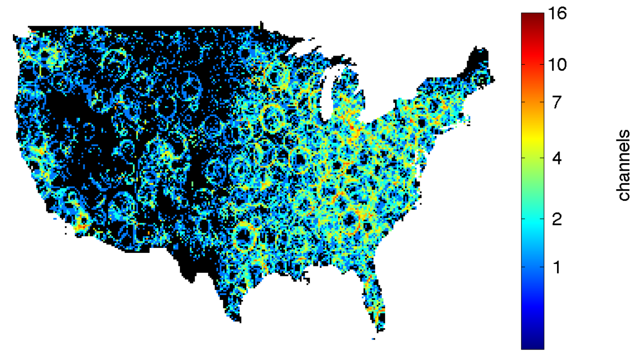

Fig. 2. Number of protected TV channels across the USA that could be lost due to interference from legally operating TV whitespace devices with one such device active for every 40 people.

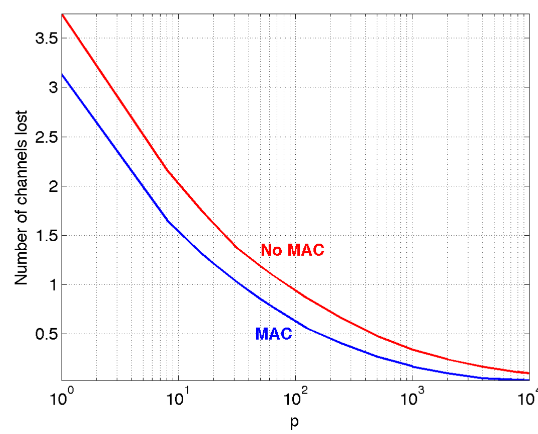

Fig. 3. The average (by population) number of protected TV channels lost because of aggregate interference as a function of the market-penetration of active fixed whitespace devices. The upper curve represents what would happen if this particular fraction of devices were ordered to transmit and simply obeyed the order as long as the database authorized them to transmit. The lower curve further assumes that the devices obey a MAC protocol that prohibits two of them from simultaneously transmitting within 200 meters of each other.

One might reasonably be skeptical as to whether so many devices could ever be simultaneously active at full power. We address the spirit of this concern in more detail within Section IV, but for now, just assume that these TV whitespace devices are attached to hackable sources/sinks of information (i.e. anything with a networked computer in it). Then even if the device itself is unhackable in terms of breaking the FCC rules, a malicious hacker or terrorist organization could inflict a distributed denial-of-service attack on broadcast television

[2]Throughout, we are simplifying the analysis. We assume that the TV receiver is omnidirectional and just set the noise figure to 0 dB. If the actual TV receiver had directional gain towards the TV tower and was pointed away from the secondary users, the loss of TV reception from the aggregate interference would be significantly less. Even on balance, increasing the noise figure while simultaneously increasing the directionality is a win from a protection point of view to the extent the TV tower and the secondary users are in opposite directions and one experiences a directionality gain while the other suffers a loss.

(one of our major sources of emergency information) and presumably also on cable systems that rely on cable head-ends to pick up the signals. All they would have to do is to tell these devices to transmit information all the time.

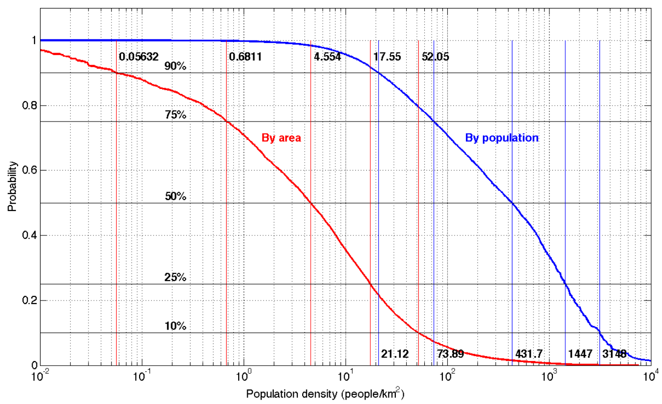

Fig. 4. Population density varies greatly across the United States: more than 10 percent of the population live in areas with more than 20 people per square kilometer and all but 90 percent live in areas of less than 3000 people per square kilometer.

The most straightforward solution to this problem would be to attempt to correct the value of the separation-margin based on some sort of expected device density — make things more conservative. However, as we see in Figure 4, the population density can vary by two orders of magnitude between the 10th and 90th percentiles. Since the secondary device density will likely be proportional to the population density, clearly such a naive approach will not work without imposing a ridiculously paranoid level of conservatism. In densely populated areas, this might very well eliminate the few whitespace channels that are available.

But some insight can be gained by thinking about how the United States could respond to such a terrorist denial-of-service attack. Would we have to reach every person and tell them to turn off their whitespace devices? No. We would instead coordinate among the whitespace database providers and tell them to start kicking users off the system by telling them that they are not allowed to operate on those channels. We would kick devices out until we had again restored functional broadcast TV service across the nation. **To be resilient to such a nationwide attack, the database systems must have a way of monitoring and controlling the density of active transmissions.**

III. Power Density

Given that we now know that the database system must have the capability to control the density, it makes sense to use this even in non-emergency scenarios. Rather than have a fixed maximum transmission power, it makes more sense to regulate the power density instead. After all, it is the power density of emissions that causes aggregate interference.

The simplest option here is to keep the idea of a fixed separation margin and just replace the fixed maximum transmission power with an enforced fixed power density throughout the United States. This would ensure that the primaries are sufficiently protected while still allowing for the existence of secondary users. However, it inherently would require secondary users to be able to either modulate their power or their duty cycle so as to guarantee that they stay within their allocated power-density footprint.

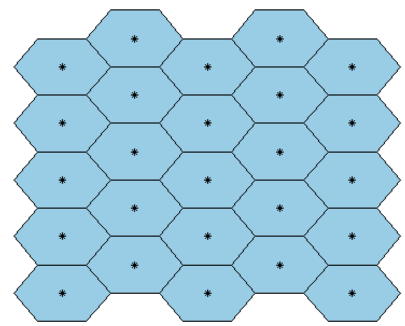

Fig. 5. To translate from a continuous power density to discrete transmitters, each transmitter is given a footprint (here it is hexagonal). The transmitter's power is then power/area $*$ area/transmitter. Note that we continue to use this hexagon model throughout. Each cell is assumed to be hexagonal with a transmitter at its center. In the cellular model, the users are placed uniformly at random within each cell. In the hotspot model, the users are placed uniformly at random within 100 meters of the access-point in the center.

When a secondary transmitter enters the system, it is given a power based on its footprint such that they collectively obey the power density rule. (In reality, this will happen in a way that also involves MAC protocols and we defer that discussion to Section IV.) This is illustrated in Figure 5 where each hexagon's worth of power density determines how much power the single transmitter within that hexagon can use to transmit.

When we take a rule of this spirit[3] and apply it to the continental United States for every channel using the existing 14.4km separation distance (as well as the existing FCC adjacent-channel separations), we get a particular power-density profile across the country. Dividing this power up among transmitters spaced according to the local population density automatically gives higher powers to transmitters in rural areas as compared to urban areas where the more closely spaced transmitters have to share power to avoid creating too much aggregate interference. The resulting downlink data rate in a cellular model is shown in Figure 7 for cells sized to hold 2000 people. The rate that any individual would get can be obtained by dividing by 2000. All of the protected TV receivers are safe since the received aggregate interference can never exceed thermal noise.

[3]We set a single safe power-density on a per-channel basis. This was done using the complete set of all television towers together viewed in isolation in a manner similar to that used in Figure 1. The allowed power-density was calculated for each of them assuming the 14.4km separation and then the minimum value was chosen as safe (*i.e.* causing in aggregate no more than thermal noise at protected TV receivers). The resulting power-densities were around -13dBm per square-kilometer for the low VHF channels, -3 dBm per square-kilometer for the high VHF channels, and then smoothly ranged from 7-11 dBm per square kilometer for the UHF channels. As might be expected, the allowed power-density must drop as the wireless propagation gets better.

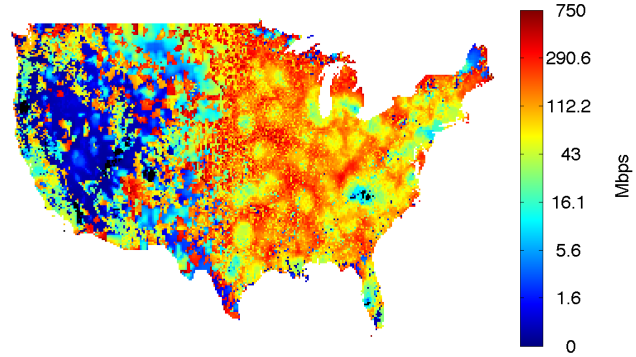

Fig. 6. Hypothetical aggregate downlink data rates available within a cellular-style system operating under the FCC rules for TV whitespaces using fixed devices with cells sized to contain 2000 people each. This assumes TVDB transmissions with 4W EIRP. Notice how the data-rates drop in the west where the population densities are lower.

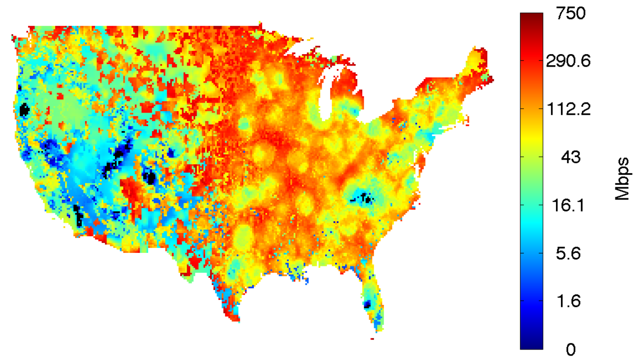

Fig. 7. The downlink data rate per cell under a model of cells sized to have 2000 people in them, and to use all available TV whitespace under *almost* the current FCC rules for fixed devices. The difference is that instead of specifying a fixed 4W for each TVBD, we have a flat power-density for each channel. Big cells can therefore use more power without causing harmful interference.

Notice that in Figure 7, the very-rural areas of the American West still see their data rates collapse, just as they would under the FCC's 4W EIRP limit model, whose cellular-style downlink rates are shown in Figure 6. This is because although the economically viable larger cells do get to use higher transmission powers, this is not enough to overcome the longer distances that must be overcome to reach everyone. Intuitively it makes sense that we should significantly increase the power density for those secondary users that are far away from all TV towers. However, if we were to do so without decreasing the power density that nearby secondary users get, it would be a disaster for the protected TV channels.

Section V sets up a simple analytic toy model that shows how we might be able to navigate the tradeoff intelligently.

IV. Will Medium-Access-Control (MAC) Protocols Save the Day on Their Own?

A. MAC Protocols and activity-density

Before seeing how to safely give higher power densities to low population-density areas, it is useful discuss MAC protocols and see where they fit in. The idea of a MAC protocol is to share spectrum fairly among active nearby devices. Although there are many variations in the details, the basic concept is that nearby wireless devices take turns using the channel instead of using it at the same time. If we assume that devices will transmit at a certain fixed power, this turn-taking has the effect of capping the power density.

The key question is whether this cap will be good enough. To get an answer, we need to first understand the nature and purpose of this turn-taking. The MAC protocols we have today among unlicensed devices are generally not imposed upon them by government regulations, rather they emerge from voluntary standards bodies or vendor designs. This is a good indication that they exist for relatively self-interested reasons. The basic principle behind a MAC protocol is that if I harmfully interfere with you and you harmfully interfere with me, then we would both be better off taking turns rather than interfering with each other [15]. At some large enough geographic separation, a system will not consider transmissions by another device as being harmful interference — at least not harmful enough to be worth losing out on an opportunity to transmit in exchange for less interference. It is this range that effectively determines the activity-density cap imposed by the MAC protocol.

For example, suppose that we are looking at a MAC protocol that will impose turn-taking on any two transmitters that are within 200 meters of each other. This means that there will be at most one active transmitter within a disk of radius 100 meters, giving an activity-density cap of one active transmitter per 0.0314 square kilometers. The second line plotted in Figure 3 shows that such a MAC protocol, while helping, is not enough to stop the loss of television reception at protected receivers.

So what sets the turn-taking range of a MAC protocol? The key concept here is the SNIR — the received signal power to thermal noise plus interference power ratio. For those unfamiliar with it, the importance of SNIR will become clearer after reading Section V. For now, it suffices to say that wireless communication requires a certain minimal SNIR to be able to support a given spectral efficiency of bits/sec/Hz [16]. The numerator is the received signal power and this largely depends on the range from the transmitter to its own intended receiver. The interference power comes from the aggregate interference of all wireless transmitters that are not within the MAC-exclusion radius. The consequence is that the MAC-exclusion radius required to maintain that SNIR will shrink with the intended range of the transmitter to its own receiver.

For a relatively short-range wireless protocol like one aimed at hotspot-style usage, the 200 meter exclusion range is quite reasonable. Since supporting hotspots is clearly a potentially socially useful goal, we cannot rule this out from self-interested devices. Even when the MAC-exclusion radius is defined implicitly by a carrier-sense threshold (what signal power we should listen for before declaring the spectrum locally unused and free for us to use), the optimal threshold ends up behaving the same way [17]. A similar effect occurs with frequency reuse in cellular-style systems — in general, modern systems tend to reuse the same frequencies within each cell. This turns out to be the best thing to do to improve the area-spectral-efficiency [18].

B. MAC protocols combined with power control

We've seen that self-interested interference-sensitive wireless devices will not necessarily want to hold down their activity-density. Instead, activity-density will be a function of the local population density, market penetration, and the desired communication range. But what about the power density? Would they be averse to adjusting their transmit powers to compensate? After all, the adopted FCC rules themselves mandate "*TVBDs shall incorporate transmit power control to limit their operating power to the minimum necessary for successful communication. Applicants for equipment certification shall include a description of a device's transmit power control feature mechanism.*" [2, pg 63]

To understand why devices should be able to control their power density, it is worth exploring a very simple toy model. Consider just three nodes arranged equally spaced along a line: our transmitter, our receiver, and the dominant interferer not excluded by the MAC protocol. Let the spacing between any two closest nodes be d. Notice that this toy builds in the key insight of activity density — that it will scale with the communication range. Let's see how the required power changes as d varies.

Suppose that the path-loss function is $d^{-\alpha}$ so that the signal power from a transmitter with power P received at range d is $Pd^{-\alpha}$. Further, suppose that the dominant interferer and the transmitter always use the same power P. If there were no background noise, the received signal power to interference power ratio would always be 1 regardless of the range d. We could reduce the transmit power as much as we wanted. But this is not realistic. Let's pick units so that thermal noise has unit power. In this case, it is easy to see that the SNIR would be $\frac{1}{1+\frac{1}{Pd^{-\alpha}}}$. Here, $Pd^{-\alpha}$ represents what the SNIR would have been had there been no interferers at all. Suppose that the minimum SNIR for "successful communication" were $\frac{1}{2}$. In this case, the system would be willing to dial the transmit power down as the range d shrank as long as $P \geq d^{\alpha}$.

To see the net effect on power-density, consider a unit of area and suppose that we fill it with K simultaneous users so the activity density is K. The communication-range distance d is thus dropping proportional to $\frac{1}{\sqrt{K}}$. The power transmitted by any individual transmitter could drop as $K^{-\frac{\alpha}{2}}$ without impeding successful communication. The resulting power-density would be K times this, and would thus scale as $K^{1-\frac{\alpha}{2}}$. Real-world path-loss exponents α always tend to be larger than 2 and so this says that it is in principle possible to have the net power density *drop* with increased population-densities even as the activity density increases.

This is a hopeful sign, but there are two main challenges:

- The above back-of-the-envelope analysis only shows proportionality and scaling behavior. There is no guarantee that the actual deployment density and target SNIR will always give rise to safe power-density levels from a primary-protection perspective.
- Transmitters might want to use more power than the minimum described here to achieve better propagation through signal fades due to obstacles, etc.

We can thus conclude that MAC protocols have natural knobs that can be used to regulate the local power density, but these knobs must both be made tunable and be set properly depending on the local context. Without a stronger regulatory mandate to do so, the pull of simplicity might argue against device manufacturers doing so. After all, existing MAC protocols in the ISM band do not try very hard to adapt their transmission powers to the lowest possible level given their specific situation.

Finally, there is also the possibility of future wireless systems that somehow make themselves largely immune from interference among themselves by deploying sufficiently advanced technology such as interference-alignment [19]. Having them maintain a fixed power density even as the device density increases will require this to be included as an explicitly mandated design objective.

V. Analytical Results

The goal of this section is to explore the reason why it is not possible to simultaneously give every location the power-density that it would be dreaming of. It will become clear that if the path-loss function is an inverse-power-law, then the transmit power that a distant location is dreaming of is growing as a power of its distance to the protected TV receiver. As we have seen in the discussion above in Section IV, for interference-sensitive users, the SNIR saturates with transmit power. This means that there is very little benefit to going to higher and higher transmit powers. Consequently, in this section we consider a model without interference where increasing the transmit power does allow one to get to higher and higher SNRs. Here, we exploit an information-theoretic reality — that the utility (in terms of data rate) can fundamentally only increase logarithmically in terms of the SNR. This means that at high SNR, one could give a user a large constant fraction of their dreamed-of utility at only a very small fraction of their desired transmit power. We will see how this suggests that a universally approximately-optimal power-control rule might exist.

For simplicity, consider the toy one-dimensional world in Figure 8. Let the protected radius r_p be so the TV SNR in a clean channel is 18 dB. The decodability constraint is that the SNR at r_p must remain above 15 dB even with interference from secondary users. So secondaries may cause no more interference than thermal noise (T). A standard inverse-power-law propagation model $r^{-\alpha}$ ($\alpha > 1$) is used.

To see what power density each location is dreaming of, consider the following simple power rule for a secondary S at

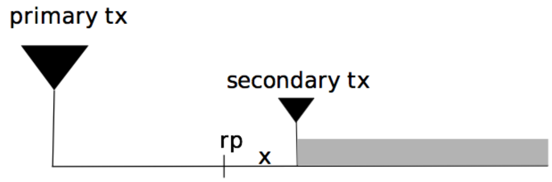

Fig. 8. Toy one-dimensional world

distance x from a point at r_p (as in Figure 8). According to S, there are no other secondaries nearer to r_p than himself *and* all the secondaries further from r_p are all using the same[4] power density as S. Knowing that they all combined may cause only T interference, S calculates the amount of power-density $P_{dream}(x)$ they he may use:

$$T = P_{dream}(x)\int_x^\infty r^{-\alpha}dr = P_{dream}(x)\frac{x^{-\alpha+1}}{\alpha-1}$$
$$\implies P_{dream}(x) = T(\alpha-1)x^{\alpha-1}$$

While S believes that the primary's SNR condition will be met under this scheme, it is easy to see why he is wrong if everyone is dreaming of their own power. Even if we exclude any users ϵ away from r_p, under this rule, the aggregate interference, N, to a point at r_p is

$$\int_\epsilon^\infty P_{dream}(r)r^{-\alpha}dr = \int_\epsilon^\infty T(\alpha-1)\frac{1}{r}dr = \infty \quad (1)$$

and thus the primary's SNR condition is not actually met. It doesn't matter how large we make the exclusion-zone ϵ, the integral will still diverge.

In order to preserve the primary's SNR while giving the secondary users' reasonable rates, we consider another rule: if a secondary user occupying unit "area" (length in this 1-dimensional toy model) would have received rate $R_{dream}(x)$ in a clean channel by using the power $P_{dream}(x)$, he now chooses power $P_{new}(x,\gamma)$ such that $R_{new}(x,\gamma) = \gamma R_{dream}(x)$. The $0 \le \gamma < 1$ can be reduced until the SNR requirement is met for the primary receiver located at r_p.

To calculate the rate, we need to choose a communication range. Let this be d. The Shannon formula for capacity tells us: rate $= \log_2(1 + \frac{\text{signal power}}{\text{noise power}})$.

[4]Some judgement must be applied regarding the issue of what power a location can *legitimately* dream of. After all, if we allowed locations to dream that everyone else was not allowed to transmit, then they would get much higher powers. But this is an illegitimate dream because it is predicated on special treatment for them in particular. Our approach is to let them lobby for the setting of parameter values like 4 W and 14.4km on a per-channel basis. The rules will then apply "fairly" to everyone — meaning that those even further away from the protected receivers get to use at least as much power as anyone closer in. Whenever one is thinking about universal competitive optimality, it is important to choose the reference-class that we compete against in a reasonable way [20]

$$\begin{aligned} R_{dream}(x,d) &= \log_2\left(1+\frac{P_{dream}(x)d^{-\alpha}}{T}\right) \\ &\approx \log_2\left(\frac{P_{dream}(x)d^{-\alpha}}{T}\right), \end{aligned}$$

$$\begin{aligned} R_{new}(x,d,\gamma) &\approx \gamma\log_2\left(\frac{P_{dream}(x)d^{-\alpha}}{T}\right) \\ &= \log_2\left(\frac{T^\gamma(\alpha-1)^\gamma x^{\gamma(\alpha-1)}d^{\alpha(1-\gamma)}d^{-\alpha}}{T^\gamma}\right) \end{aligned}$$
$$\implies P_{new}(x,\gamma) = \left(T(\alpha-1)^\gamma d^{\alpha(1-\gamma)}\right)x^{\gamma(\alpha-1)}$$

The important thing is that the scaling with distance x from r_p is now slower than $x^{\alpha-1}$. So integrals like (1) representing aggregate interference will converge now.

To ensure that the TV receiver at r_p is protected, we adjust γ so that the aggregate interference is less than or equal to T.

$$\begin{aligned} T &\ge \int_\epsilon^\infty P_{new}(r,\gamma)r^{-\alpha}dr \\ &= T(\alpha-1)^\gamma d^{\alpha(1-\gamma)}\int_\epsilon^\infty r^{-(\gamma+(1-\gamma)\alpha)}dr \end{aligned}$$
$$\Leftrightarrow 1 \ge \frac{(\alpha-1)^{-(1-\gamma)}}{1-\gamma}d^{\alpha(1-\gamma)}\epsilon^{-(\alpha-1)(1-\gamma)}$$

Note that convergence is guaranteed by the conditions $\alpha > 1$ and $0 \le \gamma < 1$. However, we still assume that the interference from the secondaries begins ϵ away from the point at r_p. This is analogous to the FCC's $r_n - r_p$. We see the best choices for γ as a function of ϵ in Figure 9.

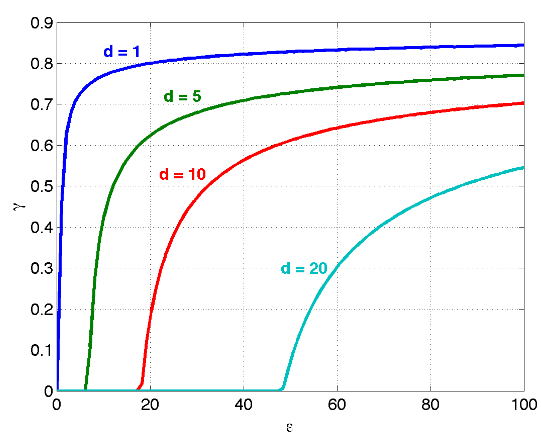

Fig. 9. For the 1-dimensional toy: the approximation-factor γ as a function of the radius sacrificed ϵ. The effect of the communication range d here is to require larger radii to be sacrificed and also to reduce the approximation factor obtained. This is implicitly a tradeoff between the additive and multiplicative approximation qualities.

By construction, the primary user's SNR requirement is satisfied. The only question remaining is whether ϵ can be given a satisfactory interpretation. Those fixed-link users beyond ϵ will receive a constant fraction γ of the rate that they would have achieved. But those within ϵ receive nothing at all!

From a multiplicative perspective, this seems like a very large sacrifice. However, an additive perspective on approximation makes it easier to interpret ϵ.

Imagine that we are considering a user that is located just outside ϵ. Such a user's dreamed-about signal-to-noise ratio would be $\frac{P_{dream}(\epsilon)d^{-\alpha}}{T}$, which corresponds to a rate of at most $\log(1 + \frac{P_{dream}(\epsilon)d^{-\alpha}}{T})$. But notice that since $\alpha > 1$, $P_{dream}(x, \gamma)$ is zero at $x = 0$ and is continuous around that point. This means that for small ϵ or large d, even the dreamed-of rate is very small, say less than β bits/sec/Hz. From an additive-approximation perspective, giving up such a small rate is not a large sacrifice — and in practice it will not be significant if there is another channel available here on which much larger powers are allowed. This means that we have obtained a (β, γ) approximately-optimal power-control rule: the data rate is off by no more than a factor of γ or an additive amount of β from what we would have achieved had we managed to tailor the whitespace rule specifically for us.

The toy model here immediately generalizes to a two-dimensional world with the only real difference is that the condition $\alpha > 1$ is replaced with $\alpha > 2$ and the single integrals become double integrals in polar coordinates. Since integration can be done numerically, rules of the same spirit can be obtained for more realistic propagation models such as the ones that the FCC uses while considering TV whitespaces.

VI. The Power Rules Applied to the United States

To see whether the approximately-universal power-density-control rule framework of Section V can actually deliver gains in a practical scenario, we apply it to the United States following an overall approach similar to that taken[5] in [14]. First, we compute a dream scenario wherein each location in the United States dreams about how much power they could use on a particular channel if locations closer than them to any given TV transmitter on that channel were not permitted to use that channel and all those further away did not get any more power. Each TV tower's protected radius is the same as that specified by the FCC rules and the limit on the power densities is calculated using the constraint that the aggregate interference should not exceed thermal noise for any protected receiver. Adjacent channel exclusions are imposed by assuming a 40dB attenuation[6] of adjacent-channel interference. The resulting dreamed-of power density is highly variable as can be seen in Figure 10. The resulting dreamed-of data rates in the cellular model are also quite high as seen in Figure 11. In lower-population-density areas the advantage of the higher powers is quite dramatic as can be seen by comparing with Figures 6 and 7.

[5]There are three significant differences: (A) We take the harmonic mean of the data rates to the potential users within our single cell or hotspot rather than simply assuming that the user is going to be located at the edge of our cell or out at the maximum range. (B) because the power density here is allowed to vary, we are more careful and generally account for self-interference coming from the users distributed like those in neighboring pixels of the map in addition that coming from users like those within our own pixel. This primarily affects low-to-moderate population-density areas where the users within our own pixel do not dominate. (C) for very low population density regions where a single cell is assumed to span many pixels, the data rate is smoothed across that larger region by choosing the median of the data rate that would be provided to us assuming that our serving base-station was in any of those pixels.

[6]This was computed based on the difference between the separation margin chosen by the FCC for co-channel vs adjacent-channel protection. Presumably that difference is only because the TV receivers can themselves reject interference coming from adjacent channels by a certain number of dBs.

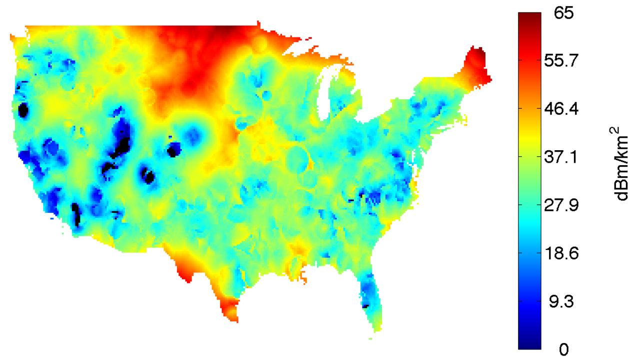

Fig. 10. The average (across whitespace channels) power density that every location is dreaming about if that location could choose a geographic-separation on a per-channel basis and a power-density on a per-channel basis so as to maximize the power available to it. Points far from TV towers dream about being able to use more power and telling those that are closer than them to be quiet. If everyone were to follow their dream, there would be a collapse of TV availability since the aggregate interference would overwhelm the system.

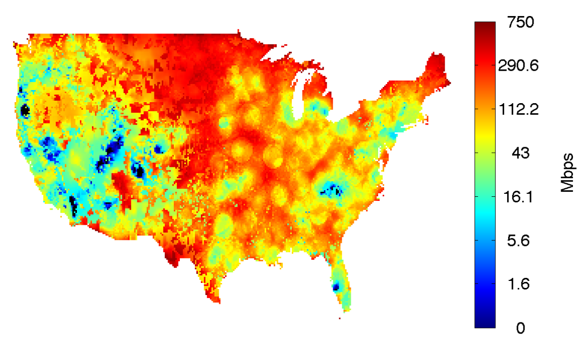

Fig. 11. The impossible cellular data rates that result from the dreamed power map with dreamed-up interference assuming that everyone around them is exactly like them. This assumes one cell for every 2000 people.

In applying the new rules of Section V, three parameters need to be chosen. The first is the "quality threshold" β by which a channel is deemed to be so poor that it is not worth having at all. Throughout this paper, the threshold is always set to 0.5 bits/sec/Hz assuming a channel that only faces thermal noise and interference from the TV signal itself. This allows us to set the allowed power to zero very close to the protected receivers since even the dreamed-of power

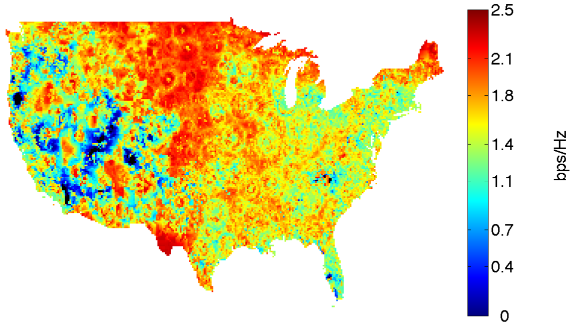

Fig. 12. The dreamed about average spectral efficiencies within the cellular model of Figure 11.

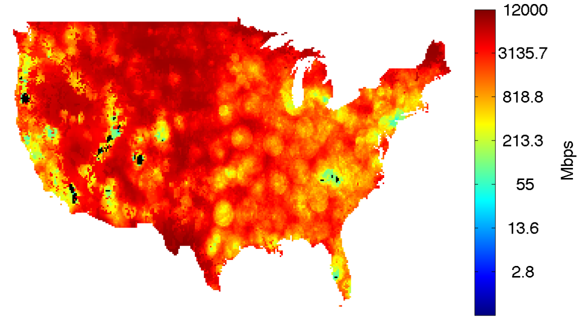

Fig. 13. The hotspot downlink data-rates that result from a candidate approximately optimal power control rule that is implicitly aimed at fixed interference-free links of 100 meters range. There is one access point for every 2000 people and the receivers do face interference from the other access points.

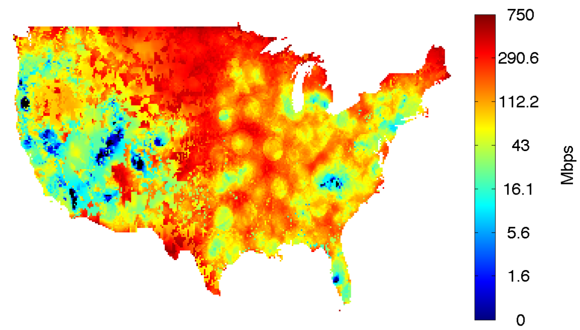

Fig. 14. The aggregate cell downlink data-rates that result from a candidate approximately optimal power control rule that is implicitly aimed at fixed interference-free links of 100 meters range. Each cell contains 2000 users and faces the actual interference that comes from everyone else.

there does not result in even a moderate-quality link. The second is the target range used to calculate the utility of the channel. The third is the area that is assumed to be represented by one link — this allows the translation from the power density to the hypothesized transmit power in the rule. The approximation quality γ is maximized subject to the constraint of not violating the aggregate interference constraint.

For a first rule, we just set the range d to be a fixed 100 meters and set the area used to calculate the power to be a fixed 0.0314 square kilometers. Implicitly, this is a rule that seems to be aimed at an advanced point-to-point deployment wherein the devices are able to eliminate all interference. By construction, the rule will deliver approximately optimal (the ratio ranges from 0.5 to 0.7) performance for such users — but how does it do for more realistic hotspots that actually do face interference from other secondary users and have a density of access points that scale with the local population density? This question is answered in Figures 13 and 15. It is clear that essentially all locations get very close to the best hotspot rate that they could have hoped for. The ratios themselves also confirm our intuition that universal rules aimed at interference-free users will tend to perform even better for interference-prone users — since the benefit of higher powers tends to saturate for the interference-prone users.

Doing well for the hotspot model is good, but our real motivation for allowing more power in rural areas was to improve performance in a cellular-style model of communication where the range could be longer. It was for this model that Figure 7 showed that the rates in very rural areas were quite low while Figure 10 shows that many of those locations are dreaming of much higher rates. The cellular performance of this new approximately-universal rule is illustrated in Figures 14 and 16. The ratio is quite high for the highly populated parts of the country and stays high for the moderately populated parts as well. It is doing much better than Figure 7 (notice the improvements in Montana and Wyoming) but does suffer from poor performance relative to the dreamed-of cellular rates in the very sparsely populated areas.

Intuitively, this is partially coming from the discrepancy between the universal rule's implicit focus on short-range communication (100m) and the reality of much longer-range communication (tens of kilometers) in these very sparsely populated areas. To better deal with such cases, we can set both the communication range and the assumed footprint of the transmitter to be inversely proportional to the population density in the natural way. The resulting rule behaves somewhat better for the cellular model in many of the rural areas while not doing too much worse in populated areas. This is shown in Figure 17. Even the performance of the hotspot model is not degraded that much as is shown in Figure 18.

However, there remain certain remote regions of the American West where the performance under the new rule remains poor. Figure 12 shows the main reason why this is happening. These are largely regions where the spectral efficiency is quite poor, even in the completely unfeasible dreamed-of powers. They are so sparsely populated that the signals simply

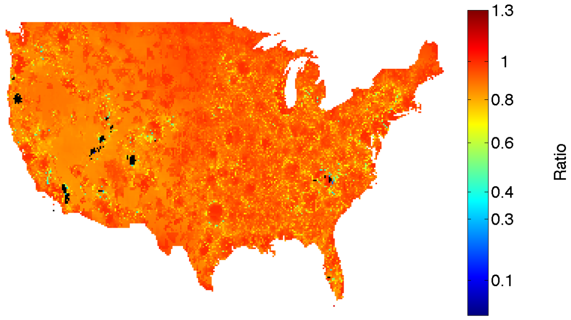

Fig. 15. The ratio of the rates actually delivered by the universal rule represented in Figure 13 to those legitimately dreamed about by each location.

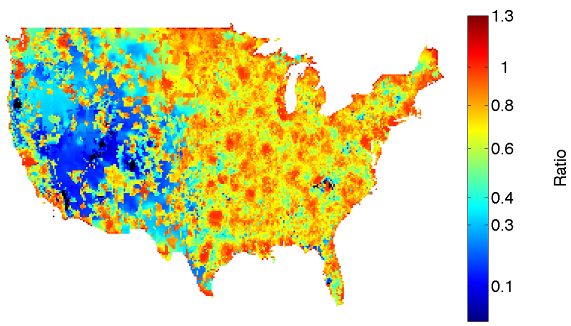

Fig. 16. The ratio of the cellular rates actually delivered by the universal rule represented in Figure 14 to those dreamed about.

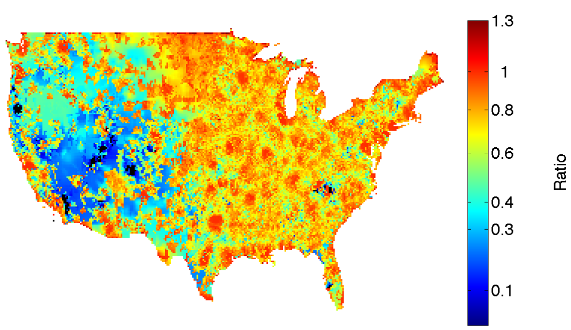

Fig. 17. The ratio to the dream of the cellular rates delivered by a second candidate approximately universal rule that ties the assumed communication range to the local population density.

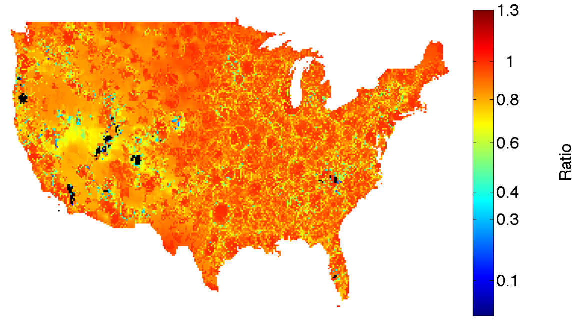

Fig. 18. The ratio to the dream of the hotspot rates delivered by the second candidate universal rule that ties the assumed communication range to the local population density.

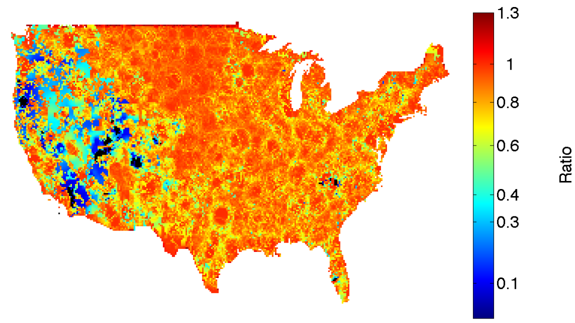

Fig. 19. What happens when we increase the penetration of cell towers to be one for every 125 people to the ratio to the dream of the cellular rates delivered by the second candidate approximately universal rule that ties the assumed communication range to the population density.

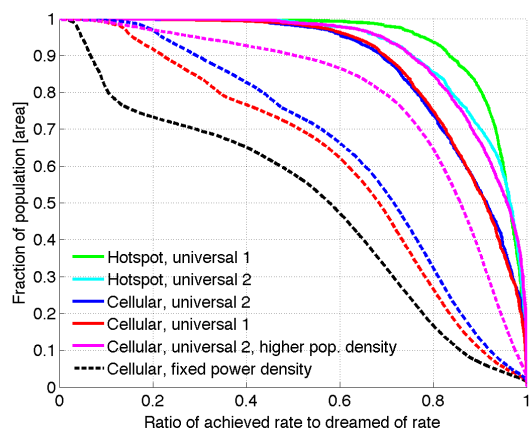

Fig. 20. CCDF of the ratios of achieved rates to dreamed-of rates, viewed from both the perspectives of population (solid lines) and area (dashed lines). Universal 1 is the universal rule that implicitly targets a fixed range of 100 meters while Universal 2 is the rule that lets the targeted range vary with the local population density. The fixed power density rule is the one from Figure 7, and the “higher pop. density” curve is the one from Figure 19.

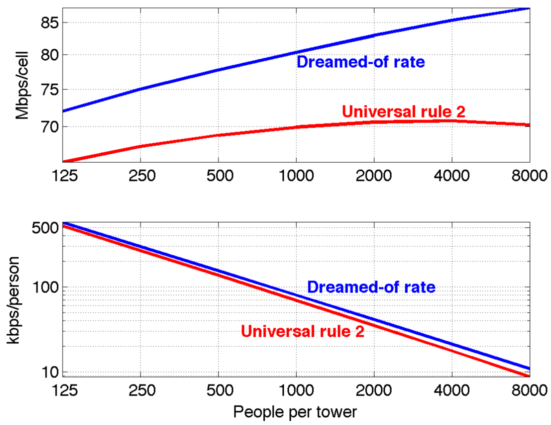

Fig. 21. Effect of p on rates per person and per cell

cannot propagate far enough to reach 2000 people without also causing interference to protected televisions.

The optimism overall comes from looking at the distribution of the rate-ratios (relative to their dreamed-of-rates) by population and area in Figure 20. We notice that most people are getting a reasonably high fraction of the downlink rates that they would have gotten with the rules that were tailor-made for their specific scenario. With the second rule that adjusts to local population densities, both the hotspot model and the cellular model are able to give more than ninety percent of the population more than seventy percent of the rates that they could have dreamed about!

The only way to serve the essentially-written-off regions with very sparse populations in a cellular-style model is to add more towers. This is shown in Figure 19 by putting in more towers so that they only need to serve 125 people. The improvement is also visible in the appropriate CCDF line of Figure 20.

However, the real advantage of decreasing cell sizes is going to be felt everywhere — fewer people will have to share the data capacity of the cell. This is shown in Figure 21. Notice how the average downlink capacity of an cell can actually shrink a little as the cell gets smaller. But this slight reduction due to more interference from nearby cells is more than compensated for by the gains achieved by having to divide the cell's downlink capacity among fewer people.

VII. Conclusions and Future Work

Under the current FCC rules, TV receivers are not as protected as they are currently assumed to be. Aggregate interference should not be ignored, but fortunately, the database-oriented approach seems to be patchable to regulate aggregate interference rather than transmit power. On the regulatory side, this has several consequences, each of which deserves to be explored in more detail:

- The FCC's power-control mandate for devices should be taken seriously in certification to make sure that devices have appropriately tunable MAC-protocol knobs and respond to power-density commands from the databases.
- There is likely a need for multiple databases to coordinate locally with each other to make sure that they are not overselling the density in any given area. Devices following carrier-sense MAC protocols might end up locally coordinating with each other to some extent anyway, but devices that do not carrier-sense and instead do time-division to stay within their allocated density might need much more supervision.
- Intellectually, the same arguments made here in the context of protecting TV users essentially show that traditional unlicensed use with simple per-device power-constraints and no band-manager or admission-control is not really compatible with primary protection elsewhere. The power-density approach taken here is fundamentally more compatible with the property-rights model proposed by deVany *et al* [21], and in fact, probably any reasonable spectrum property rights model.

The further exploration in this paper of the prospects for universal approximately-optimal power-density control rules is clearly only a first step. However, it is a quite promising step since it shows that it is possible to have a single rule that does deliver approximately-optimal performance for different applications for most of the population. Much remains to be done. For instance, there is the issue of antenna height. Clearly it is not transmission power alone but rather the interference caused to primary receivers which matters. There exist many HAAT (height above average terrain) and transmission power combinations which will respect the primary receivers. For simplicity we have mostly assumed a constant height of 30 meters in this paper but in reality a good rule should be adjustable for tower height.

Furthermore, the actual universally-fair power/admission-control rule need not be explicitly specified in terms of power density the way that is done here. In fact, it might be possible to view certain universally-fair power/admission-control rules at the level of power densities to be the emergent wide-area behavior resulting from the operation of finer-scale power/admission-control rules that target fairness [22]–[26]. Figuring out the right interface between such local rules and a global rule is an open question.

From a policy point of view, approximately-optimal universal power-control rules seem to have multiple advantages, all of which need to be explored more carefully and quantified in the future.

- They more closely follow the spirit of light-handed regulation since they let the market decide which use scenarios should be developed further.
- Besides the increased technical compatibility with property rights (and thus markets) mentioned above, the increased flexibility of use better allows these bands to nurture technical and application innovations as they develop, possibly for transition to other licensed bands later. We know that successful markets require low transaction costs and liquidity [27], [28]. The effect here is of allowing users to better discover their own demand curves since they will already have some spectrum for whatever kind of application they want to run and they

merely need to see how much they are willing to pay for more of the same. Without being able to run the same kind of applications, they are forced to extrapolate with all of its attendant risks.
- The workload on database providers would significantly increase beyond the simple am-I-in-or-am-I-out yes/no service mandated by the current FCC rules. However, this approach suggests that providers could be given a fraction of the interference temperature (perhaps based on the lowest bid) that they could charge users for premium access on.[7] This creates a potential market opportunity for a new kind of semi-private managed commons [29] within the TV whitespaces themselves. This would allow use to be, at least in part, rationally rationed when congestion occurs. By making only a fraction of the allowed power density subject to market rules, innovative new spectrum uses would never be completely shut out of spectrum access. Meanwhile, the resulting higher prices for premium access could enable other purely private bands to better respond to the demand by helping offload mature users onto their bands.

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